# Tutorial 4

Preliminary:

$$s_{\overline{n}|i} = \frac{(1+i)^n - 1}{i}, \quad a_{\overline{n}|i} = \frac{1-v^n}{i}$$

$$\ddot{s}_{\overline{n}|i} = \frac{(1+i)^n - 1}{d}, \quad \ddot{a}_{\overline{n}|i} = \frac{1-v^n}{d}$$

(Perpetuities: The infinite period annuity ($n \to \infty$) is called a perpetuity.)

$$a_{\overline{\infty}|i} = \lim_{n\to\infty} a_{\overline{n}|i} = \lim_{n\to\infty} \frac{1-v^n}{i} = \frac{1}{i}. \qquad \ddot{a}_{\overline{\infty}|i} = \lim_{n\to\infty} \ddot{a}_{\overline{n}|i} = \lim_{n\to\infty} \frac{1-v^n}{d} = \frac{1}{d}.$$

2.2.5.

$j$ is 6-month interest rate, $d_j$ is discount rate. $d_j = \frac{j}{1+j}$,

1. $\ddot{a}_{\overline{\infty}|j} = 20. \Rightarrow \frac{1}{d_j} = 20 \Rightarrow j = \frac{1}{19}.$

2-year rate $i = (1+j)^4 - 1 = (1+\frac{1}{19})^4 - 1$

X $\ddot{a}_{\overline{\infty}|i} = 20 \Rightarrow X = 20 d_i = 20 \cdot \frac{i}{1+i} = 20 \cdot \frac{(1+\frac{1}{19})^4 - 1}{(1+\frac{1}{19})^4} = 3.71.$

2.2.6.

Assume Sally's monthly payment is $K$, monthly rate is $i$

$10.000 = K\, a_{\overline{60}|i}$.

Saving account: $j^{(12)} = 6\%$, $j = 0.5\%$, monthly rate

yield rate: $k^{(2)} = 7.45\%$, $k = 3.725\%$ semiannual rate.

Accumulate Value for saving account: $K s_{\overline{60}|0.5\%}$

yield rate is define $\frac{\text{Investment value}}{\text{initial investment}} = (1+k)^n$ $\Rightarrow$ $10,000(1+3.725\%)^{10} = K$ $\Rightarrow$ $K = 206.62$

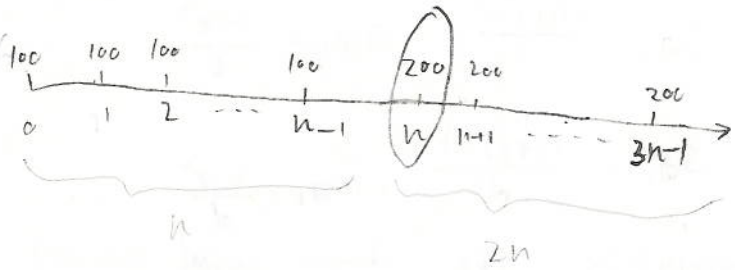
$10,000 = 206.62\, a_{\overline{60}|i} \Rightarrow i = 0.0073$ monthly rate. $\Rightarrow$ nominal rate $i^{(12)} = 12i = 0.088.$

2.2.9.

Seasonal rate $i = \frac{i^{(4)}}{4} = 0.04$. $n$ is number of deposits

$100\ \ddot{s}_{\overline{n}|0.04} = 200\ \ddot{a}_{\overline{2n}|0.04}$

$$100 \cdot \frac{(1+0.04)^n - 1}{d_{0.04}} = 200 \cdot \frac{1 - v_{0.04}^{n}}{d_{0.04}}$$




$$v^{-n} - 1 = 2(1 - v^n)$$

$$v^n(v^{-n} - 1) = 2v^n(1 - v^n)$$

$$1 - v^n = 2v^n - 2v^{2n}$$

$$2v^{2n} - 3v^n + 1 = 0$$

$$(2v^n - 1)(v^n - 1) = 0$$

$$2v^n = 1$$

$$v^n = \frac{1}{2}$$

$v = \frac{1}{1.04}$

$$\left(\frac{1}{1.04}\right)^n = \frac{1}{2}$$

$$(1.04)^n = 2$$

$$n = \frac{\ln 2}{\ln 1.04} = 17.7$$

18.

Problem Set 4: 2.2.3. 2.2.4. 2.2.8. 2.2.10. 2.2.16. 2.2.13. 2.2.20. (2.2.5, 2.2.6. 2.2.9)

2.2.3.


annual rate is 7%, quarter rate is $(1+i)^4 = 1+7\% \Rightarrow i = (1.07)^{\frac{1}{4}} - 1 = 0.017$

$450 s_{\overline{40}|i} = Y \ddot{a}_{\overline{41}|7\%} \Rightarrow Y = 9872.$

(end, beginning)

2.2.4

4-year rate: $j = (1+i)^4 - 1$

$100 \ddot{s}_{\overline{10}|(1+i)^4-1=j} = X$

$100 \ddot{s}_{\overline{5}|(1+j)^4-1=j} = \frac{X}{5}$ $\Rightarrow j = 0.3195, \quad X = 6195.$

2.2.8.

Annual rate: $i = (1 + \frac{i^{(12)}}{12})^{12} - 1 = 0.094$, monthly rate: $j = \frac{i^{(12)}}{12}$.

$100 \ddot{s}_{\overline{12n}|j} + 1000 s_{\overline{n}|i} \geq 100{,}000.$ n is number of month.

$\Rightarrow n \geq 18.3, \quad n = 19.$ Apr 30th, 2013.

2.2.10.

Account A: $1000 \ddot{s}_{\overline{10}|0.05} (1.05)^{n-10} \leq 500 \ddot{s}_{\overline{n}|0.05} \Rightarrow n \geq 30.32.$

Jan 1st, 2015.

2.2.16.

11 month payments are 1, last month is 2, which is equivalent to 12 month of payment 1 plus annual payment 1.

monthly rate is j, annual rate is $i = (1+j)^{12} - 1 = j \cdot \frac{(1+j)^{12}-1}{j} = j s_{\overline{12}|j}$.

month present value: $\frac{1}{j}$, annual present value: $\frac{1}{i}$, $(\frac{1}{i} + \frac{1}{j}) = \frac{1}{j}(1 + \frac{1}{s_{\overline{12}|j}})$.

2.2.13.

$12{,}000 = 592.15\, a_{\overline{24}|j} = 426.69\, a_{\overline{36}|j}$.

$\Rightarrow j = 0.0140$. $i^{(12)} = j \times 12 = 0.168$.

$12{,}000 = K \cdot a_{\overline{48}|j} \Rightarrow K = 345.02$.

2.2.20.

(a) $1 \cdot s_{\overline{n}|3\%} \cdot (1+4\%)^n + 1 \cdot s_{\overline{n}|4\%} \geq 100$. $\Rightarrow n \geq 22.4$. $\Rightarrow n = 23$.

(b) $1 \cdot s_{\overline{n}|3\%} (1+4\%)^n + 1 \cdot s_{\overline{n}|4\%} \geq 100$ $\Rightarrow n = 22$. (the equation is 106.618 when $n = 22$).